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THE INTERACTION OF FRICTIONAL HEATING AND WEAR AT A TRANSIENT SLIDING CONTACT†

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A solution of the axisymmetric contact problem for a half-space is obtained taking into account transient heat generation due to friction and wear, assuming a quadratic variation of the normal displacements along the radial coordinate. The problem is reduced to a non-linear Volterra integral equation in the dimensionless radius of the contact area. Asymptotic solutions of this equation are constructed for short and long periods of time, and a numerical algorithm is also developed for investigating it in the general case. The proposed mathematical methods enable the effect on the dimensions of the contact region of two processes of opposite kind, namely, frictional heating and wear, to be investigated.

The interaction of frictional heating and wear under steady thermal conditions at a contact was investigated in [1, 2]. Transient heat generation was considered in [3] taking the wear between the contacting surfaces in the plane formulation into account. An algorithm for solving the corresponding axisymmetric contact problem is developed below.

1. Suppose an elastic axisymmetric massive body (an elastic punch) with shear modulus μ and Poisson's ratio ν, is pressed with a force P and slides with a velocity V, sufficiently small to enable inertial effects to be neglected, over the surface of a rigid half-space in the direction of the x-axis (Fig. 1). Wear of the base of the punch then occurs, accompanied by heat generation due to friction in the contact region. We will assume that the surface of the first body is heat conducting, while the second body is a perfect heat insulator. All our subsequent discussions will be carried out in a system of x, y, z coordinates connected with the punch. When solving the problem we will consider it in section as a half-plane, the boundary of which is slightly curved (R_0 is the radius of curvature).

Starting from the equations of uncoupled thermoelasticity, we can write the contact condition between the interacting bodies

$$u_z^e + u_z^T + u_z^w = \Delta(t) - r^2/(2R_0) \quad (r \leqslant a(t), t \geqslant 0) \tag{1.1}$$

Here $u_z^e(r, t)$ are the elastic displacements of the boundary points of the punch in the direction of the z-axis, $u_z^T(r, t)$ are the normal temperature displacements, $u_z^w(r, t)$ are the vertical displacements due to wear and $\Delta(t)$ is the approach of bodies 1 and 2 as rigid wholes. It is assumed that u_z^T, u_z^w and Δ are commensurable with u_z^e, and thus we can formulate the boundary conditions with respect to the undeformed surfaces of the contacting bodies, assumed in the linear theory of elasticity, i.e. relation (1.1) holds when $z = 0$.

At the initial instant of time the function u_z^e and the contact pressure $p(r, t)$ in the Hertz theory approximation, are given by the formulae [4] ($r \leqslant a(0)$)

$$u_z^e = \frac{3P[2a^2(0) - r^2]}{16\theta a^3(0)}, \quad p = \frac{3P\sqrt{a^2(0) - r^2}}{2\pi a^3(0)}, \quad \theta = \frac{\mu}{1-\nu} \tag{1.2}$$

The first equation in (1.2) is obtained on the assumption that the displacement u_z^e due to the action of the normal stresses is considerably greater than that due to the shear forces. An estimate of the error resulting from this assumption can be found, for example, in [5].

We will approximate the sum of the thermal displacements of the surface of the punch due to heating by the heat flux

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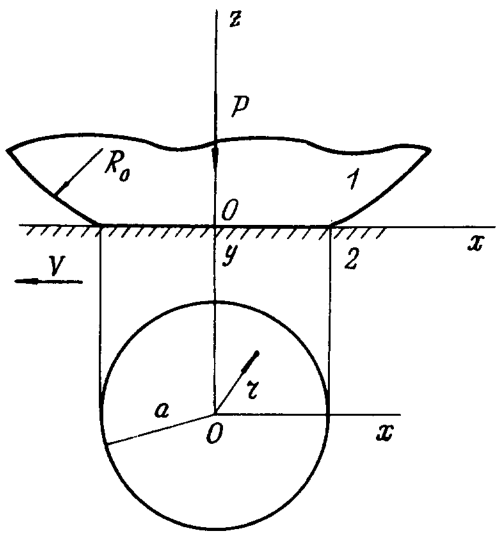

Fig. 1.

$$q(r, t) = fVp(r, t) \quad (r \leqslant a(t), t \geqslant 0) \tag{1.3}$$

(f is the coefficient of sliding friction) and its normal displacements due to wear at each instant of time $t \geqslant 0$ by the quadratic relation

$$u_z^T(r,t) + u_z^w(r,t) = C_0(t) + C_2(t)r^2 \quad (r \leqslant a(t)) \tag{1.4}$$

Then, using (1.4), we can write boundary condition (1.1) in the form

$$\begin{aligned} &u_z^e(r,t) = D_0(t) - D_2(t)r^2 \quad (r \leqslant a(t)) \\ &D_0(t) = \Delta(t) - C_0(t), \quad D_2(t) = C_2(t) + (2R_0)^{-1} \\ &C_j(0) = 0 \quad (j = 0, 2) \end{aligned} \tag{1.5}$$

and, consequently, when finding $u_z^e(r, t)$ and $p(r, t)$ $(t > 0)$ we can use Hertz's formulae. Hence, comparing the coefficients of r^2 in expressions (1.2) and (1.5) we have

$$D_2(t) = 3P[16\theta a^3(t)]^{-1} \quad (t \geqslant 0) \tag{1.6}$$

Introducing into the second equation of (1.2) the value of the radius of the contact area $a(t)$ from (1.6) we obtain

$$p(r,t) = 8\pi^{-1}\theta D_2(t)\sqrt{a^2(t) - r^2} \quad (r \leqslant a(t), \ t \geqslant 0) \tag{1.7}$$

Hence, it follows from relations (1.3), (1.6) and (1.7) that to determine the contact pressure $p(r, t)$, the frictional heat flux $q(r, t)$ and the radius of the contact area $a(t)$ it is sufficient to know the form of the function $D_2(t)$.

Note that under steady conditions of heat generation, the radius of the contact zone can be calculated using the formula [6]

$$a_0 = \pi K[1.566(1 + \nu)fV\alpha_T\theta]^{-1} \tag{1.8}$$

where K is the thermal conductivity and α_T is the coefficient of linear thermal expansion of the material of the punch. The quantity a_0 here is independent of the pressing force P and is the limiting value of $a(t)$ as $P \to \infty$. This limit does not exist in the purely isothermal problem.

2. Let us determine the temperature $T(r, t)$ and the normal displacement $u_z^T(r, t)$ of the surface of an elastic half-space heated by a heat flux $q(r, t)$ of the form (1.3). To do this we will use the fundamental solution [7] of the unsteady heat-conduction equation for a source of heat of constant power q^0 acting instantaneously at the point O (Fig. 1)

$$T^0(r,t)=\frac{q^0\exp[-r^2(4kt)^{-1}]}{4\rho c(\pi kt)^{3/2}}\quad (r\geqslant 0,\ t>0) \tag{2.1}$$

Here $k = K(\rho c)^{-1}$ is the thermal diffusivity, ρ is the density and c is the specific heat capacity of the material of the half-space.

The vertical displacement of the boundary of the half-space $z = 0$, free from external forces, corresponding to the thermal field (2.1) is equal to [8]

$$u_z^0(r,t)=-\frac{\alpha_T(1+\nu)q^0}{4\pi Kt}\Phi\left(\frac{3}{2};\ 2;\ -\frac{r^2}{4kt}\right),\quad (r\geqslant 0,\ t>0) \tag{2.2}$$

where $\Phi(\alpha; \gamma; x)$ is the confluent hypergeometric function.

From (2.1) and (2.2), bearing in mind the principle of superposition (the instantaneous point source method [7]), we have

$$T(r,t)=\frac{1}{4\rho c(\pi k)^{3/2}}\int_0^t\int_0^{a(\tau)}\int_0^{2\pi}q(s,\tau)e^{-X^2}\frac{sd\varphi dsd\tau}{(t-\tau)^{3/2}} \tag{2.3}$$

$$u_z^T(r,t)=-\frac{\alpha_T(1+\nu)}{4\pi K}\int_0^t\int_0^{a(\tau)}\int_0^{2\pi}q(s,\tau)\Phi\left(\frac{3}{2};\ 2;\ -X^2\right)\frac{sd\varphi dsd\tau}{t-\tau},$$
$$X^2=\frac{r^2-2rs\cos\varphi+s^2}{4k(t-\tau)},\quad r\leqslant a(t),\quad t>0 \tag{2.4}$$

When formulating contact problems of the theory of elasticity with wear the law of wear is most often taken in the form [9]

$$u_z^w(r,t)=k_w\int_0^t p(r,\tau)d\tau\quad (r\leqslant a(t)) \tag{2.5}$$

(k_w is the wear coefficient). Such a relationship occurs when the wear is produced by abrasive particles and in some cases of fatigue wear.

Recently more complex hereditary-ageing models [10, 11] have come to be used, which take into account the influence of after-effect

$$u_z^w(r,t)=k_w\int_0^t K_1(\tau)K_2(t-\tau)p(r,\tau)d\tau\quad (r\leqslant a(t))$$
$$K_1(\tau)=1+\alpha e^{-\beta\tau},\quad K_2(t-\tau)=e^{-\gamma(t-\tau)} \tag{2.6}$$

where α, β, γ are parameters determined experimentally for each tribocoupling. The wear law (2.6), unlike (2.5), enables processes with limited wear to be described.

Note that in expressions (2.5) and (2.6) it is necessary to follow the change in the contact area with time, and when it increases monotonically, for example, in the segment $\theta_1 \leqslant t \leqslant \theta_2$ ($\theta_1 < \theta_2$, $\theta_1 \geqslant 0$) we must put [12]: $p(r_*, t) = 0$ when $t \leqslant t_*$ ($\theta_1 \leqslant t_* \leqslant \theta_2$). Here t_* is the time taken for the boundary of the contact region to reach arbitrary points with coordinates $z = 0$, $r_* = a(t_*)$.

We substitute relations (1.3) and (1.7) into (2.4) and (2.5). Using (1.8) we can write the last formulae in the form

$$u_z^T(r,t) = -\frac{8k}{1.566\pi a_0}\int_0^t \int_0^{A(t,\tau)} D_2(\tau)\sqrt{(A^2-S^2)g(t-\tau)}SF_0(R,S)dSd\tau \tag{2.7}$$

$$u_z^w(r,t) = \frac{8\theta V k_w}{\pi}\int_0^t D_2(\tau)\sqrt{a^2-r^2}d\tau \tag{2.8}$$

Here

$$A^2 = a^2/g, \quad R^2 = r^2/g, \quad S^2 = s^2/g, \quad g(t-\tau) = 4k(t-\tau) \tag{2.9}$$

$$F_0(R,S) = \int_0^{2\pi} \Phi\left[\frac{3}{2};\ 2;\ -(R^2 - 2RS\cos\varphi + S^2)\right]d\varphi$$

To calculate the quadrature in (2.9) we will expand the confluent hypergeometric function in series

$$\Phi\left(\frac{3}{2};\ 2;\ -X^2\right) = \sum_{i=0}^{\infty} \frac{(2i+1)!!(-X^2)^i}{(2i)!!(i+1)!} \quad (X^2 < \infty)$$

Using the integral [13]

$$\int_0^{2\pi}\left[1 - 2\frac{R}{S}\cos\varphi + \left(\frac{R}{S}\right)^2\right]^i d\varphi = 2\pi\sum_{j=0}^{i}(C_i^j)^2\left(\frac{R}{S}\right)^{2j}$$

we obtain

$$F_0(R,S) = 2\pi\sum_{i=0}^{\infty}\frac{(2i+1)!!(-S^2)^i}{(2i)!!(i+1)!}\sum_{j=0}^{i}(C_i^j)^2\left(\frac{R}{S}\right)^{2j} \tag{2.10}$$

We will now introduce the function

$$F(R,A) = \int_0^A \sqrt{A^2-S^2}F_0(R,S)SdS \tag{2.11}$$

Substituting (2.10) into (2.11) and taking into account the value of the integral [13]

$$\int_0^A \sqrt{A^2-S^2}S^{2i-2j+1}dS = \frac{(2i-2j)!!}{(2i-2j+3)!!}A^{2i-2j+3}$$

we will have

$$F(R,A) = -2\pi A\sum_{j=0}^{i}\sum_{i=j}^{\infty}\frac{(-1)^{i+1}(2i+1)!!(2i-2j)!!}{(2i)!!(i+1)!(2i-2j+3)!!}(C_i^j)^2\left(\frac{R}{A}\right)^{2j}A^{2i+2} \tag{2.12}$$

In (2.12) we have changed the order of summation with respect to i and j.
We will write the normal displacement of the surface of the punch due to wear (2.8) in the form

$$u_z^w(r,t) = \frac{8\theta V k_w}{\pi}\int_0^t D_2(\tau)a(\tau)[1 - r^2/(2a^2) + \ldots]d\tau \tag{2.13}$$

This representation is justified [14] by virtue of the observation, made above, regarding the behaviour of the contact pressure in the integrands of (2.5) and (2.6).
Now comparing the coefficients of r^2 in relations (1.4), (2.7), (2.12) and (2.13) we obtain

$$D_2(t) = \frac{8k}{1.566a_0}\sum_{i=1}^{\infty}\frac{i}{(i+1)!}\int_0^t(-A^2)^{i+1}D_2(\tau)[a(\tau)]^{-1}d\tau -$$

$$-\frac{4\theta V k_w}{\pi}\int_0^t D_2(\tau)a^{-1}(\tau)d\tau + D_2(0), \quad (D_2(0)=(2R_0)^{-1}) \tag{2.14}$$

We will change the dimensionless variables

$$\tilde{t} = 4kta_0^{-2}, \quad \tilde{\tau} = 4k\tau a_0^{-2} \tag{2.15}$$

in (2.14) and use the sum of the series

$$\sum_{i=1}^{\infty}\frac{(-A^2)^{i+1}i}{(i+1)!} = 1-(1+A^2)e^{-A^2} = B[A(t,\tau)] = \tilde{B}(t,\tau)$$

We thereby obtain a Volterra integral equation in $D_2(\tilde{t})$

$$D_2(\tilde{t}) = \frac{a_0}{0,783}\int_0^{\tilde{t}} \tilde{B}(\tilde{t},\tilde{\tau})D_2(\tilde{\tau})a^{-1}(\tilde{\tau})d\tilde{\tau} -$$

$$-a_0\kappa\int_0^{\tilde{t}} D_2(\tilde{\tau})a^{-1}(\tilde{\tau})d\tilde{\tau} + D_2(0) \quad \left(\kappa = \frac{\theta V k_w}{\pi k}\right) \tag{2.16}$$

Using the new required function $\tilde{a}(\tilde{t}) = a(\tilde{t})a_0^{-1}$ and relation (1.6), we can convert integral equation (2.16) to the form (the tilde is omitted)

$$0,783\left[\frac{1}{a^3(t)}-\frac{1}{a^3(0)}\right] = \int_0^t [B(t,\tau)-0,783]\frac{d\tau}{a^4(\tau)} \quad (t \geqslant 0) \tag{2.17}$$

If the vertical displacement $u_z^w(r, t)$ is given by the hereditary-ageing type relation (2.6) the second term in the integrand of (2.17) must be multiplied by the product $K_1(\tau)K_2(t-\tau)$.

3. We will construct a scheme for solving integral equation (2.17). To do this we split the integration interval $[0, t]$ into L parts of length $h = tL^{-1}$: $0 = \tau_0 < \tau_1 < \ldots < \tau_L = t$. In each section $\tau \in [\tau_{j-1}, \tau_j]$ ($j = 1, 2, \ldots L$) we approximate the required function $a(\tau)$ by constants $a(t_j) \equiv a_j$ ($t_j = \tau_j - h/2$). Using the first formula of (2.9) and (2.15) with $\tau \in [\tau_{j-1}, \tau_j]$ we obtain

$$A^2 = a_j^2(t-\tau)^{-1}, \quad d\tau = 2a_j^2A^{-3}dA \tag{3.1}$$

Integral equation (2.17) is then transformed into a recurrent algebraic relation which enables the value of $a(t_L)$ to be determined from the preceding values $a(t_j)$ ($j = 1, 2, \ldots, L-1$)

$$\frac{1}{a^3(t_L)}-\frac{1}{a^3(0)} = -\frac{1}{0.783}\sum_{j=1}^{L-1}\frac{2a_j^2A_{jL}+0.783\kappa h}{a_j^4}$$

$$A_{jL} = \int_{\alpha_{1j}}^{\alpha_{2j}}\frac{B(A)}{A^3}dA, \quad \alpha_{mj}^2 = \frac{a_j^2}{\hat{t}_{mj}} \quad (m=1,2) \tag{3.2}$$

$$\hat{t}_{mj} = [L-j+\tfrac{1}{2}(-1)^m]h$$

We start the calculations using Eqs (3.2) at $L = 1$; in this case $a(t_1) = a(0)$, which corresponds to the radius of the contact area of the isothermal Hertz problem. If the surface of the elastic punch is initially not curved, i.e. $a(0) \to \infty$ and $D_2(0) = 0$, integral equation (2.16) has the trivial solution of $D_2(t) = 0$ ($t > 0$), which corresponds to a uniform pressure distribution over the surface of the half-space. The influence functions A_{jL} in (3.2) were found using a software package [15].

Figure 2 shows the change in the dimensionless radius $a(t)$ of the contact area in the case of abrasive wear (2.5). The initial value of $a(0)$ was assumed to be 5. Curves 1–3 were calculated for $\kappa = 0$, 0.5 and 1.5, respectively. If there is no wear ($\kappa = 0$) the function $a(t)$ is a linearly decreasing function (see (3.3) below) and for sufficiently large values of the time reaches a value of $a_\infty = 0.783a_0$, which is the solution

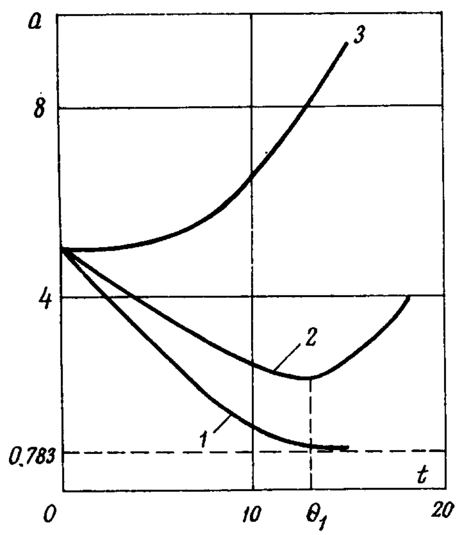

Fig. 2.

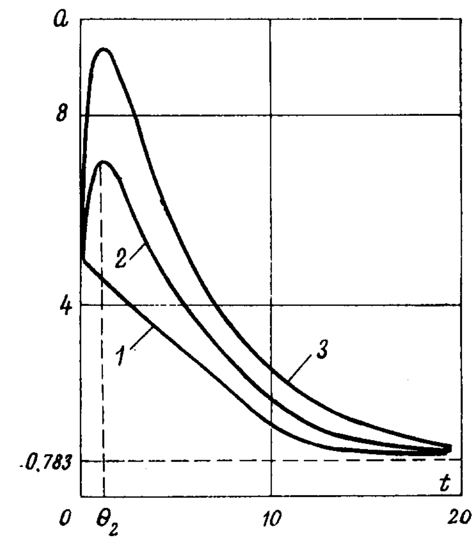

Fig. 3.

of the problem under steady heat-generation conditions. A numerical analysis shows that in the range $9 < \kappa < 1.277$ there is always a value $t = \theta_1$, beginning from which the contact area will increase monotonically. For $\kappa > 1.277$ the function $a(t)$ is a monotonically increasing function over the whole period of operation of the coupling considered.

The radius of the contact area behaves somewhat differently in the hereditary-ageing model of wear (2.6) (Fig. 3). The results presented here were obtained for the following values of the parameters: $\alpha = 40$; $\beta = 1$; $\gamma = 0.5$. As above, curves 1–3 correspond to $\kappa = 0, 0.5$ and 1.5. The presence at the initial stage of the contact interaction when $\kappa > 0$ of a peak in the values of $a(\theta_2) > a(0)$, after reaching which $a(t)$ must reach the asymptote a_∞, is characteristic for this form of wear.

We will now construct the asymptotic solutions of integral equation (2.17) for short and long times. In the first version, as follows from (3.1), $A \gg 2$, whence $B(A) \approx 1$. The initial integral equation is then written in the form

$$\frac{1}{a^3(t)} - \frac{1}{a^3(0)} = (1.277 - \kappa)\int_0^t \frac{d\tau}{a^4(\tau)} \quad (t \geqslant 0)$$

and its solution is the linear function

$$a(t) = a(0) - {}^1/_3(1.277 - \kappa)t \tag{3.3}$$

Relation (3.3) confirms the previous conclusions that the dimensionless radius of the contact area may be either a monotonically decreasing function or a monotonically increasing function depending on the value of the coefficient κ for small values of the time. In the first case frictional heat generation predominates at the contact, while in the second wear predominates.

Using (2.15) and the notation introduced above, we can convert (3.3) to dimensional form

$$a(t) = a(0) - \frac{4kt}{3a_0}(1.277 - \kappa)$$

Hence it follows that the rate of change of the radius of the contact region is inversely proportional to the radius of the contact part a_0 (1.8) under steady heat-generation conditions.

Suppose now that $t \to \infty$ in (2.17). Putting $a(\infty) = a_\infty = \text{const}$ and using the first equation of (2.9), we obtain

$$d\tau = 2A^{-3}(a_\infty / a_0)^2 dA$$

Substituting the last identity into the integrand of (2.17) we will have

$$0.783\left[1-\frac{a_\infty^3}{a^3(0)}\right]=2\frac{a_\infty}{a_0}\int_0^\infty B(A)\frac{dA}{A^3} \tag{3.4}$$

The integral on the right-hand side of (3.4) is equal to 0.5, and, consequently, we obtain the following algebraic equation for determining a_∞

$$0.783a_0\{1-[a_\infty a^{-1}(0)]^3\}=a_\infty \tag{3.5}$$

If $a_\infty a^{-1}(0) \ll 1$ in (3.5), we obtain $a_\infty = 0.783a_0$. Hence, with the above assumption that the distribution of the normal displacement of an elastic punch obeys quadratic relation (1.4), the relative error when calculating the radius of the contact area under steady and unsteady heat generation conditions may amount to 21.7%.

We can determine the temperature in the contact region from relations (1.3), (1.6), (1.7) and (2.3). Obviously the maximum values of $T(r, t)$ will be reached at the centre of the circular contact zone. When $r = 0$, by (2.15) (the tilde is omitted, as above) we obtain

$$\begin{gathered} T(0,t)=T_0\int_0^t F_1(A)\frac{d\tau}{a^3(\tau)} \quad (t>0) \\ F_1(A)=\int_0^A \sqrt{A^2-S^2}e^{-S^2}SdS, \quad T_0=\frac{P}{1,044a_0^2\sqrt{\pi}\theta\alpha_T(1+\nu)} \end{gathered} \tag{3.6}$$

Using integration by parts we can represent the quadrature $F_1(A)$ in the form

$$\begin{gathered} F_1(A)=\tfrac{1}{2}A[1-F_2(A)] \\ F_2(A)=\frac{1}{A}e^{-A^2}\int_0^A e^{x^2}dx=\frac{\sqrt{\pi}}{2}e^{-A^2}\Phi\left(\frac{1}{2};\ \frac{3}{2};\ A^2\right) \end{gathered} \tag{3.7}$$

(values of the Doson function $AF_2(A)$ are tabulated in [16]).

Figure 4 shows the evolution with time of the maximum dimensionless temperature $T^*_{\max}(t) = T_0^{-1}T(0, t)$, calculated from (3.6) and (3.7) for abrasive wear given by (2.5) of the surface of an elastic heat-conducting solid. Curves 1–3 correspond to $\kappa = 0$, 0.5 and 1.5. It can be seen that $T_{\max}$ reaches a maximum value, after which the temperature at the contact is equalized.

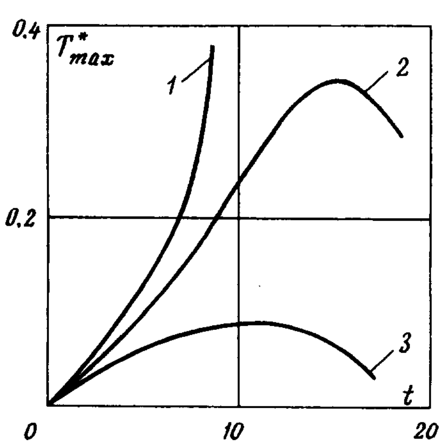

Fig. 4.

REFERENCES

1. YEVTUSHENKO A. A. and KOVALENKO Ye. V., The contact problem of the wear caused by melting of the bushing of a sliding bearing. *Prikl. Mat. Mekh.* **57**, 1, 148–156, 1993.
2. KOVALENKO Ye. V. and YEVTUSHENKO A. A., The wear of a sliding bearing taking frictional heating into account. *Treniye i Iznos* **14**, 2, 259–269, 1993.
3. AZARKIN A. and BARBER J. R., Transient contact of two sliding half-planes with wear. *Trans. ASME Ser. F. J. Tribology* **109**, 4, 598–603, 1987.
4. JOHNSON K., *The Mechanics of Contact Interaction*. Mir, Moscow, 1989.
5. ZUZNETSOV Ye. A. and GOROKHOVSKY G. A., Effect of tangential load on the stressed state of rubbing rough bodies. *Wear* **73**, 1, 41–58, 1981.
6. BARBER J. R., Some thermoelastic contact problems involving frictional heating. *Q. J. Mech. Appl. Math.* **29**, 1, 1–13, 1972.
7. CARSLAW H. and JAEGER D., *The Thermal Conductivity of Solids*. Nauka, Moscow, 1964.
8. BARBER J. R., Distortion of the semi-infinite solid due to transient surface heating. *Int. J. Mech. Sci.* **14**, 6, 377–393, 1972.
9. KRAGEL' SKII I. V. and ALISIN V. V. (eds) *Friction, Wear and Lubrication. A Handbook*, Vol. 1. Mashinostroyeniye, Moscow, 1978.
10. KUZ' MENKO A. G., Contact problems taking wear into account for cylindrical sliding bearings. *Treniye i Iznos* **2**, 3, 502–512, 1981.
11. GAVRIKOV M. V. and MAZING R. I., The hereditary-ageing model of wear and its application to problems with a monotonically growing contact zone. *Treniye i Iznos* **9**, 2, 274–279, 1988.
12. BELOKON' A. V. and VOROVICH I. I., Contact problems of the linear theory of viscoelasticity ignoring friction and cohesion forces. *Izv. Akad. Nauk SSSR. MTT* 6, 63–73, 1973.
13. GRADSHTEIN I. S. and RYZHIK I. M., *Tables of Integrals, Sums, Series and Products*. Nauka, Moscow, 1971.
14. ALEKSANDROV V. M. and KOVALENKO Ye. V., An analytic solution of the contact problem of the wear of a shaft-bushing coupling. *Treniye i Iznos* **8**, 6, 985–995, 1987.
15. PRESS W. H., FLANNERY B. P., TEUKOLSKY S. A. and VETTERING W. T., *Numerical Recipes*. Cambridge University Press, 1986.
16. KARPOV K. A., *Tables of the Function* $w(z) = e^{-z^2}\int_0^z e^{x^2}\,dx$ *in the Complex Domain*. Izd. Akad. Nauk SSSR, Moscow, 1954.

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